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## PROBLEMS AND SOLUTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

## PROBLEMS FOR SOLUTION.

## ALGEBRA.

**396. Proposed by H. E. TREFETHEN, Colby College.**Show that  $1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \sqrt{2}(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots)$ .

## GEOMETRY.

**424. Proposed by H. E. TREFETHEN, Colby College.**In a given triangle  $ABC$ , determine by geometric demonstration the point  $O$  so that the sum of the distances,  $AO + BO + CO$ , shall be a minimum.**425. Proposed by V. M. SPUNAR, Chicago, Ill.**Find the ratio of the areas,  $A_1$  and  $A_2$ , of the parabolas formed by projectiles whose ranges are the same and whose angles of projection are complements of each other.

## CALCULUS.

**345. Proposed by C. N. SCHMALL, New York City.**

"Of all the quadrilaterals which can be formed from four given sides, that which is inscriptible in a circle has the greatest area."

[From GOURSAT-HEDRICK, *Math. Anal.*, p. 133, ex. 5.]*Proposer's Remark.*—Prove this, and furthermore show that when only three sides are known, the length of the fourth side of the maximum quadrilateral is the root of a cubic equation.**346. Proposed by C. N. SCHMALL, New York City.**

Given the height of an inclined plane, to find its length so that a given force, acting on a given mass in a direction parallel to the plane, may draw it up in the shortest time.

*Note.*—This question has a practical application in the case of a truck, of known height, upon which a mass is to be loaded by means of skids.

## MECHANICS.

**280. Proposed by C. N. SCHMALL, New York City.**Given the distance  $d$  between two smooth hooks in the same horizontal line. Show that the shortest string which can form a catenary, with these hooks for points of support, is  $de$ , where  $e$  is the base of the Napierian system of logarithms.**281. Proposed by C. N. SCHMALL, New York City.** $ABC$  is a triangle inscribed in a circle, center  $O$ , and  $L, M, N$ , are the centers of gravity of the sectors  $AOB, BOC, COA$ . Show that

$$\frac{AB}{OL} + \frac{BC}{OM} + \frac{CA}{ON} = 3\pi.$$

## NUMBER THEORY AND DIOPHANTINE ANALYSIS.

**197. Proposed by E. T. BELL, Seattle, Wash.**Show that in the expansion of  $\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1$ , where  $p$  is a prime, the coefficients of the various powers of  $z$  are divisible by  $p$ . [EISENSTEIN, *Crelle*, t. 27, p. 282.]

**198. Proposed by ARTEMAS MARTIN, Washington, D. C.**

Prove that every even number is the sum of two prime numbers.

*Note.*—This problem has long been known and no proof has ever been given. [EDITORS.]

**CORRECTION.**—Number 192, incorrectly given as 188 in the June issue, page 196, should have the zeros on the right of equations (1), (2), (3) each replaced by  $\square$ , the symbol for a "square number."

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

**386. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.**

Given the sequence,  $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{5}{6}, \frac{1}{7}, \dots, u_1, u_2, u_3, \dots$ , show that

$$\left[ \frac{\sum_1^n u_i}{n} \right]_{n \rightarrow \infty} = \frac{1}{2}.$$

SOLUTION BY H. A. LEVY, Houghton, Mich.

Consider the sequence  $\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, \dots, \frac{q-1}{q}$ .

(1) Suppose  $q$  to be an odd prime. Then the quantities  $\frac{n}{q}$  ( $n = 1, 2, 3, \dots, q-1$ ) are irreducible fractions, and  $q-1$  must be an even number, say  $2k$ . Then

$$\sum_{n=1}^{q-1} \frac{n}{q} = \frac{1+2+3+\dots+(q-1)}{q} = \frac{q-1}{2} = k.$$

Hence, in such a sequence, there are  $2k$  terms, and the sum of the terms is  $k$ .

(2) Suppose that  $q$  is an even number of the form  $2^r p$ , where  $p$  is any odd prime except one. All the fractions with even numerators are then reducible, and the sequence is

$$\frac{1}{2^r p}, \frac{3}{2^r p}, \frac{5}{2^r p}, \dots, \frac{2^r p - 1}{2^r p}.$$

In this sequence, at least one term is reducible, viz.,  $\frac{p}{2^r p}$ . If there are more,

they are the terms  $\frac{p}{2^r p}, \frac{3p}{2^r p}, \frac{5p}{2^r p}, \dots$  up to the greatest odd multiple of  $p$

less than  $2^r p$ . This odd multiple must therefore be of the form  $4k+1$ . Hence an odd number of terms are reducible. Furthermore, the numerator of the last term of the sequence,  $2^r p - 1$ , is of the form  $4k+1$ , so that there is an odd number of terms in the sequence. Rejecting the reducible terms, therefore, leaves an even number of irreducible terms.

If  $r = 1$ , the number of terms is  $p-1$ , an even number. The sum of the terms is then